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AQA

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Centre number						Candidate number				

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Forename(s) ____________________

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AS MODEL SOLUTIONS

FURTHER MATHEMATICS

Paper 2 – Statistics

Thursday 17 May 2018 Afternoon Time allowed: 1 hour 30 minutes

Materials

- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \frac{3}{4}x(2-x) & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X = 1)$

Circle your answer.

[1 mark]

0 $\quad\quad \frac{1}{2} \quad\quad \frac{3}{4} \quad\quad \frac{27}{32}$

2 The discrete random variable Y has a Poisson distribution with mean 3

Find the value of $P(Y > 1)$ to three significant figures.

Circle your answer.

[1 mark]

0.149 $\quad\quad$ 0.199 $\quad\quad$ 0.801 $\quad\quad$ 0.950

$Y \sim Po(3)$

$P(Y > 1) = 1 - P(Y \leq 1)$

$= 1 - 0.199$

$= 0.801$

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3 The discrete random variable X has the following probability distribution

x	1	2	4	9
$\mathrm{P}(X = x)$	0.2	0.4	0.35	0.05

The continuous random variable Y has the following probability density function

$$f(y) = \begin{cases} \frac{1}{64}y^3 & 0 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Given that X and Y are independent, show that $\mathrm{E}(X^2 + Y^2) = \frac{1327}{60}$

[4 marks]

$E(X^2) = 1^2(0.2) + 2^2(0.4) + 4^2(0.35) + 9^2(0.05) = 11.45$

$E(Y^2) = \int_0^4 y^2\left(\frac{1}{64}y^3\right) dy = \int_0^4 \frac{1}{64}y^5 \, dy = \frac{1}{64}\left[\frac{1}{6}y^6\right]_0^4$

$= \frac{1}{64}\left(\frac{1}{6}(4)^6\right) = \frac{32}{3}$

$E(X^2) + E(Y^2) = E(X^2 + Y^2)$

$= 11.45 + \frac{32}{3}$

$= \frac{1327}{60}$

Turn over ▶

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4 The waiting times for patients to see a doctor in a hospital can be modelled with a normal distribution with known variance of 10 minutes.

4 (a) A random sample of 100 patients has a total waiting time of 3540 minutes.

Calculate a 98% confidence interval for the population mean of waiting times, giving values to four significant figures.

[4 marks]

$\bar{x} = \frac{3540}{100} = 35.4$

$Z = \Phi^{-1}(0.98) = 2.3263 \quad (4dp)$

Confidence interval: $35.4 \pm 2.3263 \frac{\sqrt{10}}{\sqrt{100}}$

: 35.4 ± 0.73564

$[34.66, 36.14]$

4 (b) Dante conducts a hypothesis test with the sample from part **(a)** on the waiting times. Dante's hypotheses are

$$H_0 : \mu = 38$$
$$H_1 : \mu \neq 38$$

Dante uses a 2% level of significance.

Explain whether Dante accepts or rejects the null hypothesis.

[1 mark]

38 is outside the range [34.66, 36.14] so Dante rejects the null hypothesis

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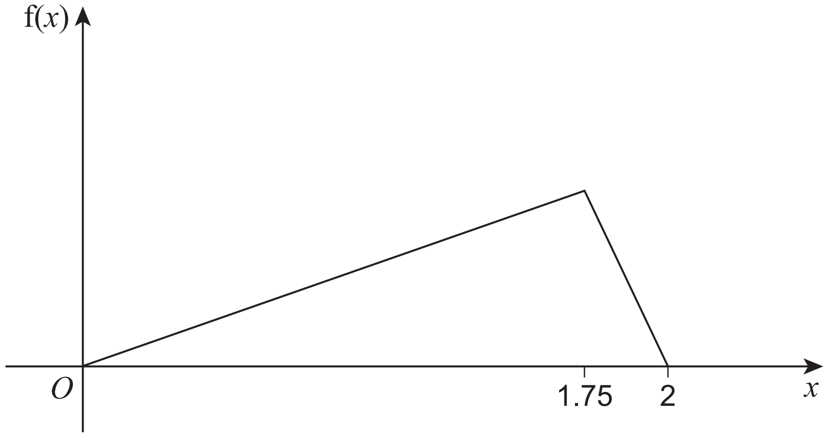
5 The diagram shows a graph of the probability density function of the random variable X.



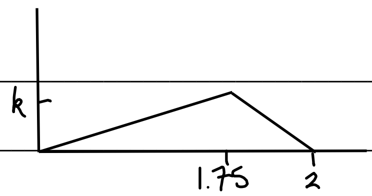
5 (a) State the mode of X.

[1 mark]

1.75

5 (b) Find the probability density function of X.

[4 marks]



Total area = 1

$bh/2 = 1$

$\frac{2k}{2} = 1$

$k = 1$

First line: $\frac{\Delta y}{\Delta x} = \frac{1}{1.75} \Rightarrow y = \frac{1}{1.75}x$

$= y = \frac{4}{7}x$

Second line: $\frac{\Delta y}{\Delta x} = \frac{-1}{0.25} = -4$

$y - y_1 = m(x - x_1)$

$y - 0 = -4(x - 2)$

$y = -4x + 8$

So the pdf is: $F(x) = \begin{cases} \frac{4}{7}x & 0 \le x \le 1.75 \\ -4x+8 & 1.75 < x \le 2 \\ 0 & \text{otherwise} \end{cases}$

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6 The discrete random variable Y has the probability function

$$P(Y = y) = \begin{cases} 2ky & y = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

Show that $\text{Var}(5Y - 2) = 25$

[6 marks]

$E(Y) = 1(2k) + 2(4k) + 3(6k) + 4(8k) = 60k$

$E(Y^2) = 1^2(2k) + 2^2(4k) + 3^2(6k) + 4^2(8k) = 200k$

Sum of the probabilities $= 2k + 4k + 6k + 8k = 1$

$\Rightarrow 20k = 1$ and thus $k = \frac{1}{20}$

So $E(Y) = 3$ and $E(Y^2) = 10$

$\text{Var}(Y) = E(Y^2) - (E(Y))^2 = 10 - 3^2 = 1$

$\text{Var}(5Y-2) = \text{Var}(5Y) = 25\text{Var}(Y) = 25$

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7 Over a period of time it has been shown that the mean number of vehicles passing a service station on a motorway is 50 per minute.

After a new motorway junction was built nearby, Xander observed that 30 vehicles passed the service station in one minute.

7 (a) Xander claims that the construction of the new motorway junction has reduced the mean number of vehicles passing the service station per minute.

Investigate Xander's claim, using a suitable test at the 1% level of significance.

[6 marks]

$H_0: \lambda = 50 \quad H_1: \lambda < 50$

$X \sim Po(50)$ where X is the number of cars passing a service station in one minute

$P(X \leq 30) = 0.002 < 0.01$

Therefore we reject H_0 meaning that there is sufficient evidence to suggest that the mean number of vehicles passing the service station has decreased

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7 (b) For your test carried out in part **(a)** state, in context, the meaning of a Type 1 error.
[1 mark]

Concluding that the mean number of vehicles passing the service station has decreased when it has not

7 (c) Explain why the model used in part **(a)** might be invalid.
[1 mark]

The mean rate might not be constant as different times of day might mean that vehicles pass the service station at varying rates. This constant mean rate is necessary for a Poisson model

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8 An insurance company groups its vehicle insurance policies into two categories, car insurance and motorbike insurance.

The number of claims in a random sample of 80 policies was monitored and the results summarised in contingency **Table 1**.

Table 1

		Number of claims				
		0	**1**	**2**	**3 or more**	**Total**
Type of insurance policy	**Car**	9	10	11	5	35
	Motorbike	19	13	8	5	45
	Total	28	23	19	10	80

The insurance company decides to carry out a χ^2-test for association between number of claims and type of insurance policy using the information given in **Table 1**.

8 (a) The contingency table shown in **Table 2** gives some of the exact expected frequencies for this test.

Complete **Table 2** with the missing exact expected values.

[2 marks]

Table 2

		Number of claims			
		0	**1**	**2**	**3 or more**
Type of insurance policy	**Car**	12.25	10.0625	8.3125	4.375
	Motorbike	15.75	12.9375	10.6875	5.625

$$E_i = \frac{\text{row total} \times \text{column total}}{\text{table total}}$$

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8 (b) Carry out the insurance company's test, using the 10% level of significance.

[8 marks]

H_0: The number of claims and type of policy are independent
H_1: The number of claims and type of policy are not independent

	0		1		≥ 2	
	O	E	O	E	O	E
Cars	9	12.25	10	10.0625	16	12.6875
Motorbikes	19	15.75	13	12.9375	13	16.3125

$$\sum \frac{(O-E)^2}{E} = \frac{(9-12.25)^2}{12.25} + \frac{(10-10.0625)^2}{10.0625} + \frac{(16-12.6875)^2}{12.6875}$$

$$+ \frac{(19-15.75)^2}{15.75} + \frac{(13-12.9375)^2}{12.9375} + \frac{(13-16.3125)^2}{16.3125}$$

$$= 3.07$$

Degrees of freedom $= (2-1)(3-1) = 2$
Critical value $= 4.605 > 3.07$

Therefore, we accept H_0. Thus there is evidence to suggest that the number of claims and type of policy are independent.

END OF QUESTIONS

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